A Comparative analysis of pressure Gradient Models for Vertical Multiphase Flow in Producing Oil Wells in Niger Delta

Suleiman O. Agboola, Dulu Appah and Oriji A. Boniface

Abstract—Gas-liquid flow commonly occurs in oil and gas production and processing structure. The prediction of pressure gradient in multiphase flow for vertical pipes is of interest for the oil industry and also a critical variable for the best design of surface facilities. Models like empirical correlations and mechanistic models are available to calculate the multiphase flow pressure gradient, holdup and phases distribution. In this study, simple and easy-to-compute pressure gradient models were developed using MS Excel and VB.Net. The models were validated with pressure gradient measured at the field. The models were developed for accurate measurement of pressure gradient in selected Niger-Delta oil wells as there is limited validity of existing correlations that are based on quality, region and scope of data upon which they were developed. Due to the extreme complexity of two-phase flow, the total pressure gradient was assumed to be dependent only on the no-slip liquid holdup (for ease of estimation) by neglecting the acceleration and frictional components since the elevation component accounts for not less than 90-99% of the total pressure gradient and neglecting the frictional term accounts for not more than 10% error. Pressure gradients got with the Orkiszewski correlation was used to model for slug flow and it gave an average percent error of 12.13% when compared with field measurement. Aziz et al correlation was used to model for both Mist and Bubble flow regimes; it gave average error percentages of 9.79% and 6.87% respectively when compared with actual field measurement. Beggs and Brill correlation was used to model for intermittent flow and it gave an average error percentage of 8.41% when compared to actual field measurement. These models would be helpful in quick accurate estimation of pressure gradient which aids; in selecting correct tubing sizes; predicting when a well quit flowing and hence predict time for artificial lift; designing artificial lift installations; determining Pwf and PIs of wells; and predicting maximum flow rates.

Index Terms—keywords: Multiphase flow, Niger Delta, Intermittent flow, Bubble flow, Slug flow, Mist flow, liquid holdup, pressure gradient

1 Introduction and Background

When two or more phases flow simultaneously in pipes, the flow behavior is much more complex than for single-phase flow. Shear stresses at the pipe walls are different for each phase as a result of their different densities and viscosities [1]. In spite of this, the ability to predict it in truly new situations is not very good. Differences are primarily as a result of the variety of flow regimes that one tries to bridge with a single correlation scheme. Another problem is the large number of dimensionless variables that are noticeably important, at least at some conditions [2]. Pressure drop of several fluids have been investigated both theoretically and experimentally by several authors [3]. Because of the highly complex and unpredictable nature of multiphase flow, most early investigators used laboratory and/or field data to develop empirical correlations for evaluating pressure drop during multiphase flow [3], [4], [5], [6], [7], [8], [9] and [10]. The validity of these empirical correlations is to some degree limited to the quality and scope of the data and type of experimental measurements used in their development. Therefore, a better approach is to attempt to model the flow system and then to test the model against actual data. Proper modeling of multiphase flow requires an understanding of the physical system [11]. Among the empirical correlations, Aziz et al correlation in [8] seems to have some theoretical justification and with some modification it could predict multiphase flow performance inside production pipelines more accurately. Pressure losses encountered during vertical flow of two phases enter into a wide range of design calculations (which may include tubing size and operating well head pressure of a flowing well). There are publications on several correlations that are used to predict pressure drop in tubings and pipes for the simultaneous, upward, continuous flow of water, oil/gas. These correlations are empirical because of the extreme complexity of multiphase flow and so there's a limited validity of the correlations based on the quality, region and scope of the data upon which they are based. Therefore some correlations fail for other applications asides performing well for cases in the range of data used in developing the correlation [12]. The aim of this study is to compare existing models for predicting multiphase flow pressure gradient in vertical flow for producing wells in the Niger-Delta province. Some authors have investigated the total pressure drop in two-phase vertical flow in pipes. Chierrci et al correlation in [3] predicted pressure drop in a two-phase vertical flow with mass transfer between the flowing phases and the average density of the flowing fluid and the friction losses were calculated according to the locally prevailing flow regime. Griffith and Wallis, and Duns and Ros correlations in [13] and [14] were used to evaluate the regions of existence of the various flow regimes. A new relationship was proposed for extrapolating at bubble Reynolds number, $(N_{Re})_b>6{,}000$ and overall Reynolds number $(N_{Re})_t>6{,}000$, the Griffith

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and Wallis correlation in [13] was used to calculate the average fluid density in the slug-froth flow regime. Their method was tested for validity on 31 actual oil well cases covering a broad range of API gravity, GOR, oil flow rate, and pressure drop. The validity of the method was shown by referring to the deviation between predicted and actual pressure drops. This showed an average error of 0.12 %, average absolute error of 4.36 % and a standard deviation 5.42 % [15]. Sandip et al. in [16] developed a new model to predict the pressure drop under foam flow conditions. By using a simple drift flux model and using some of the available literature data on foam, a foam pressure drop model was developed, and compared with 570 data points collected from many gas wells. The match between the model and the actual observations was reasonable. Khasanov et al. in [17] developed a new mechanistic model for two-phase flow in vertical and inclined pipes based on Drift-flux approach. Unlike the other mechanistic models in [18] and the unified mechanistic model, the developed model incorporated a system of nonlinear equation to solve using an explicit equation for liquid hold up prediction. The simple form of liquid hold up prediction formula enables analytical integration of pressure gradient in two phase flow along the pipe. The model was evaluated using TUFFP databank and Rosnef field data. Evaluation showed that in comparison with mechanistic models, the proposed model enables calculating pressure gradient with comparable accuracy, and less calculation resources required (Proceedings of International Oil Conference and Exhibition in Mexico IOCEM, 06/2007). Correlations used in solving multiphase flow problems are categorized into two groups, namely, empirical and mechanistic models. The empirical approach was the early method used by researchers to solve multiphase flow problems in the past. During this time the investigators employed the use of simplifying assumptions and physicals methods based on field and experimental data from the laboratory. Mechanistic model on the other hand was a later approach, based on full description of the elementary mechanism happening in multiphase flow. Empirical correlations are categorized into three groups according to [7];

Group 1

Correlations in this group do not consider slip between phases and flow regime. The three correlations in this group are as follows: Poettmann and Carpenter in [4], Baxendell and Thomas in [5], and Fancher and Brown in [6].

Group 2

For this group, slip is considered, but no flow regime is considered. Three correlations are presented in this category, they are: Hagedorn and Brown in [19], Gray and Asheim correlations.

Group 3

This category considers both slip and flow regime. Correlations in this group includes: Duns and Ros in [14], Orkiszewski in [7], Aziz et al. in [8], Chiericiet al in [3], Beggs and Brill in [9] and Mukherjee and Brill in [10].

Beggs and Brill in [10] developed a correlation to compute pressure gradient in all range of pipe inclination. They used data obtained from small-scale test facility experiment comprising of 1.0 and 1 ½ in section made from acrylic pipe, 90ft. long to develop the correlation. The fluid system used was air and water. From the experiment, they varied the gas flow rate between 0 and 300 Mscf and for the liquid flow rate, 0 and 30gal/ min. The average pressures of the system were between 35 and 95psia. Aziz et al. in [8] developed a mechanistic correlation for vertical two phase flow. The aim of their model involved the prediction of actual flow patterns based on simplified flow pattern map. The two phase properties and variables such as density, frictional factor and pressure gradient were calculated from broad equations accurate for each flow configuration. They developed new correlations for slug and bubble flow patterns. They used the Duns and Ros correlation in [14] for mist flow pattern but used interpolation method proposed by Duns and Ros in [14] for transition flow. Taitel et al in [20] developed a mechanistic model for flow pattern determination. They proposed a physical mechanism for the transition boundary in between flow patterns and modeled each transition boundary on the basis of the mechanism by which it occurs. From their model, four distinctive flow patterns, namely bubbly, churn and slug flow patterns were observed. Hassan and Kabir in [21] and [22] developed a mechanistic model for multiphase flow in vertical tube. Their model focused on transition boundaries individually. Valid criteria for each transition boundary were developed. The four different transition boundaries observed were, bubble-slug flow, the transition-dispersed bubble flow, the slug-churn flow and the transition-annular transitions boundaries. Experimental data from literature that occurred at a void fraction of 0.25 was used to determine the bubble-slug flow transition. They presumed that since a transition is a slow process, it was better to use a terminal velocity of Taylor bubbles in slug flow for determining the bubble-slug flow transition boundary. For dispersed bubbles flow pattern, the transition is ascribed to the breakdown of large bubbles in the liquid as a result of high flow rates. They used Taitel et al. equation in [20] for mixture velocity and linked it to the maximum bubble diameter possible under turbulent condition. An expression for gas superficial velocity was derived for transition to annular flow pattern because void fraction tends to unity. Ansari et al. in [18] developed a mechanistic model that predicts a transition boundary of different flow regime, pressure drop for bubble-slug flow and annular flow regime. They developed an implicit equation for calculating liquid hold up in the bubble flow pattern and derived slug flow pattern to fully developed and developing slug flow.

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2 METHODOLOGY

2.1 Model Development

A model will be developed using MS Excel, and validated using Visual Basic software. For simplicity of result, the acceleration and friction components were neglected (assumed zero) in this study. Pressure gradients will be estimated using the Orkiszewski in [7], Aziz et al in [8], and the Beggs and Brill in [9] correlations for the different flow regimes. The pressure gradients that will be got from the individual correlations will be compared to the actual pressure gradient got from the field measurement. The error percent will be estimated for each of the correlations used for the respective flow regimes.

2.2 Data Collection

Measured field data (such as oil FVF, gas FVF, oil gravity, gas gravity, liquid surface tension, viscosities of oil/gas, flowrate of oil/gas, BHFP, WHP, solution GOR, flowing temperature and pressure well depth, tubing size, measured pressure gradient etc.) were obtained from producing oil wells in the Niger-Delta.

TABLE 1 COLLECTED FLUID PROPERTIES (FP) OF THE 11 OIL WELLS

FP	1	2	3	4	5	6	7	8	9	10	11
q_l	0.8124	0.5160	0.5394	0.4250	0.7779	0..9175	0.7720	1.2283	0.2035	0.1789	0.2273
q_g	0.7919	0.8906	0.9308	0.7333	0.7616	0.4225	0.6880	0.0804	0.1315	0.1248	0.0335
API	35	35.56	35.56	35.56	40	25	36.55	36.55	40.60	42.01	37.2
B_o	1.192	1.015	1.017	1.002	1.197	1.191	1.2324	1.1729	1.0169	1.0032	1.0000
B_g	0.0078	0.0052	0.0093	0.0293	0.0091	0.0087	0.0076	1.004	0.00299	0.0033	0.0037
T	630	654	654	654	640	620	648.5	648.5	560.4	560.4	627
P	1825	2326	2080	2082	1700	2500	2781	2311	4500	4250	3800
Ö	7.95	6.98	8.24	7.16	8.41	7.38	6.79	8.84	8.41	8.41	8.41
D	0.4583	0.3298	0.3298	0.3298	0.5000	0.375	0.4178	0.5153	0.3355	0.3355	0.3355
H	9250	10289	12000	12500	8000	8400	9750	7877	12000	10000	9500
€	0.0006	0.0006	0.0006	0.0006	0.0006	0.0006	0.0006	0.0006	0.006	0.006	0.006
μ_o	1.02	0.89	0.97	3.5	0.97	1.05	2.7	0.92	0.88	1.7	4.2
μ_g	0.018	0.013	0.016	0.012	0.016	0.017	0.018	0.019	0.023	0.027	0.032
R_s	350	323.05	323.01	322.94	281	320	336.78	336.83	11363	10785	2894

3 RESULTS

3.1 Graphic User Interface of VB.Net

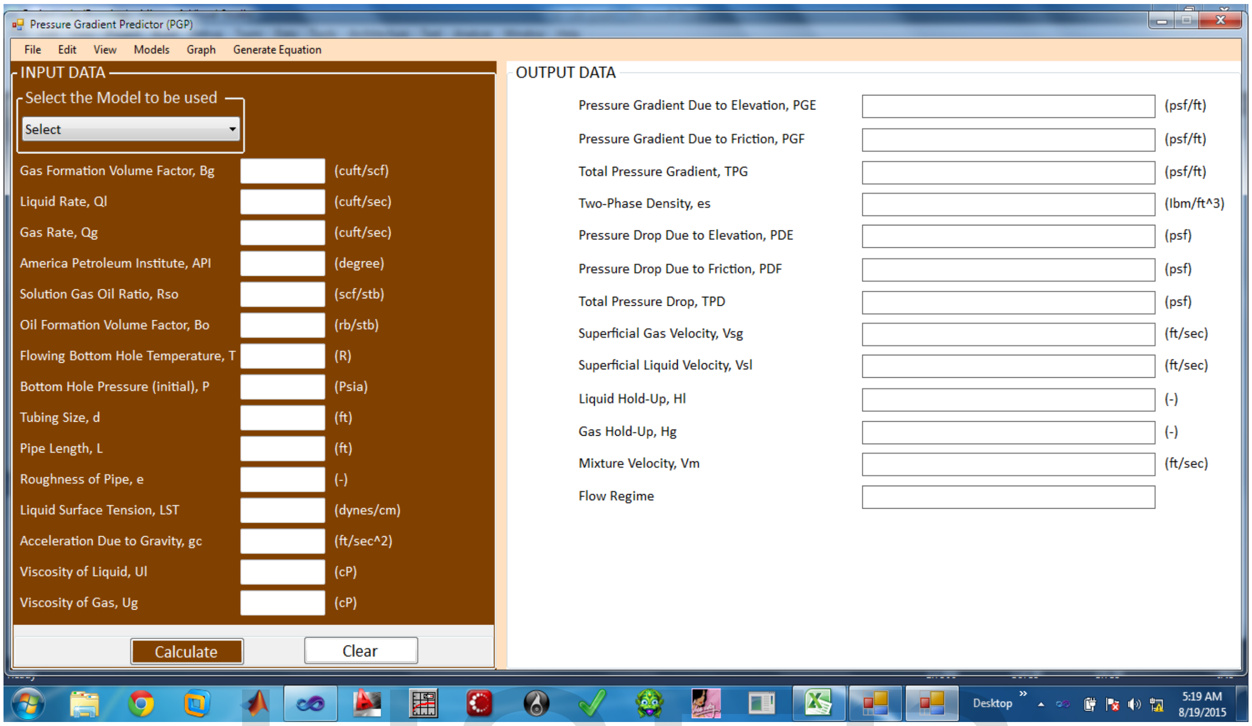

Figure 1 - The interface of the visual basic model that was developed

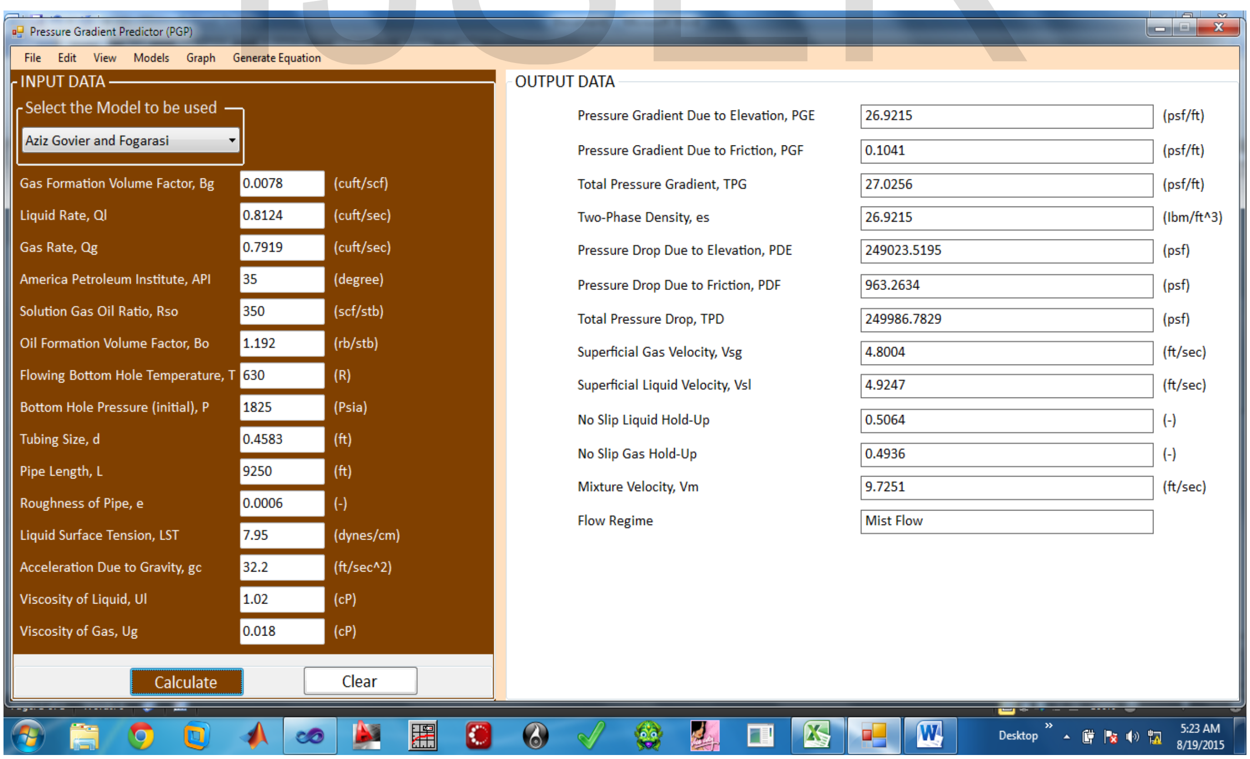

Figure 2 - analysis of well 1 with the Aziz et al model (for mist flow)

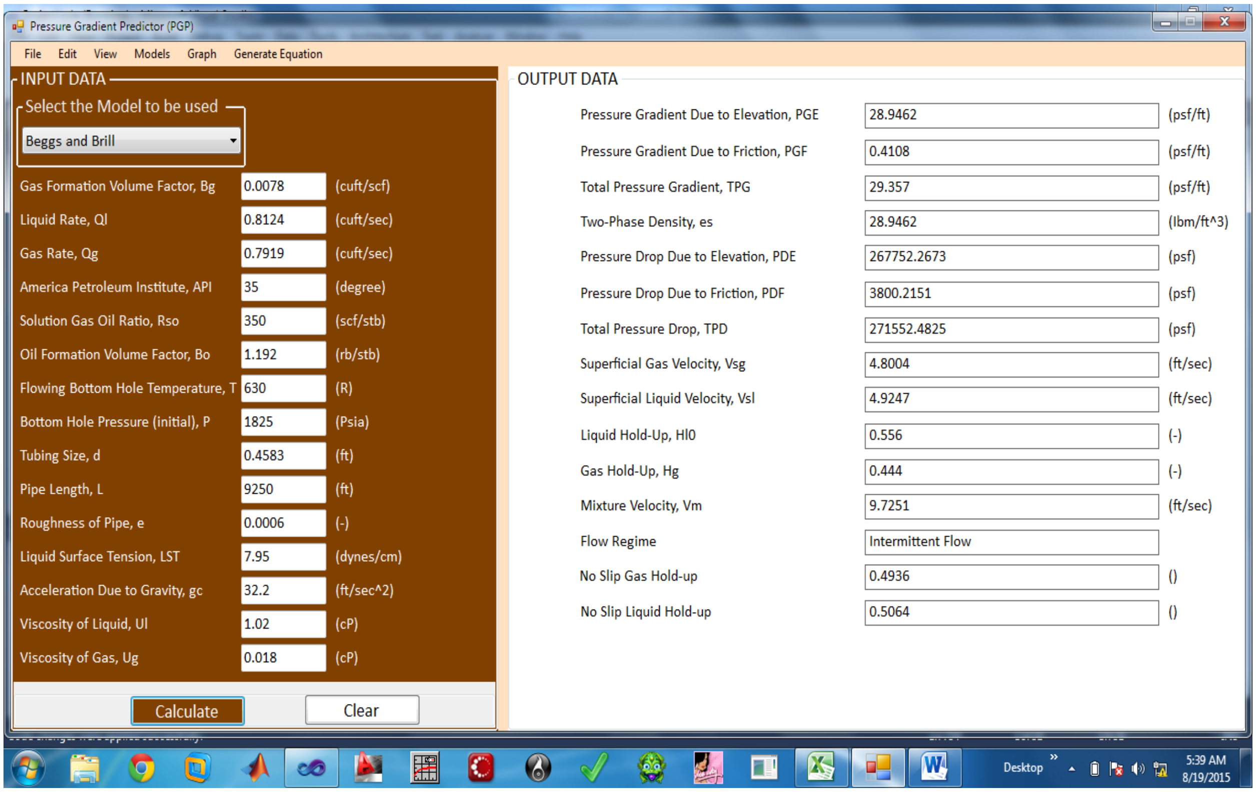

Figure 3 - analysis of well 1 with the Beggs and Bill model (for intermittent flow)

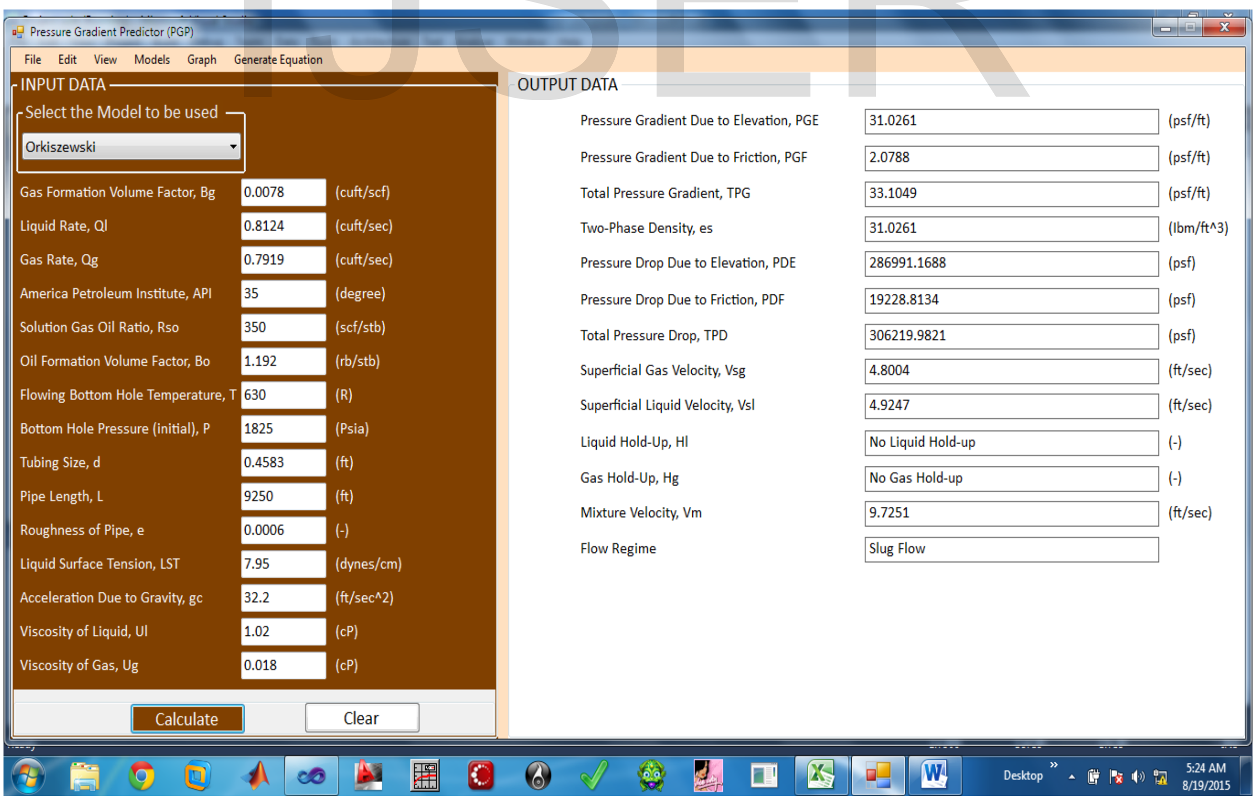

Figure 4 - analysis of well 1 with the Orkiszewski model (for slug flow)

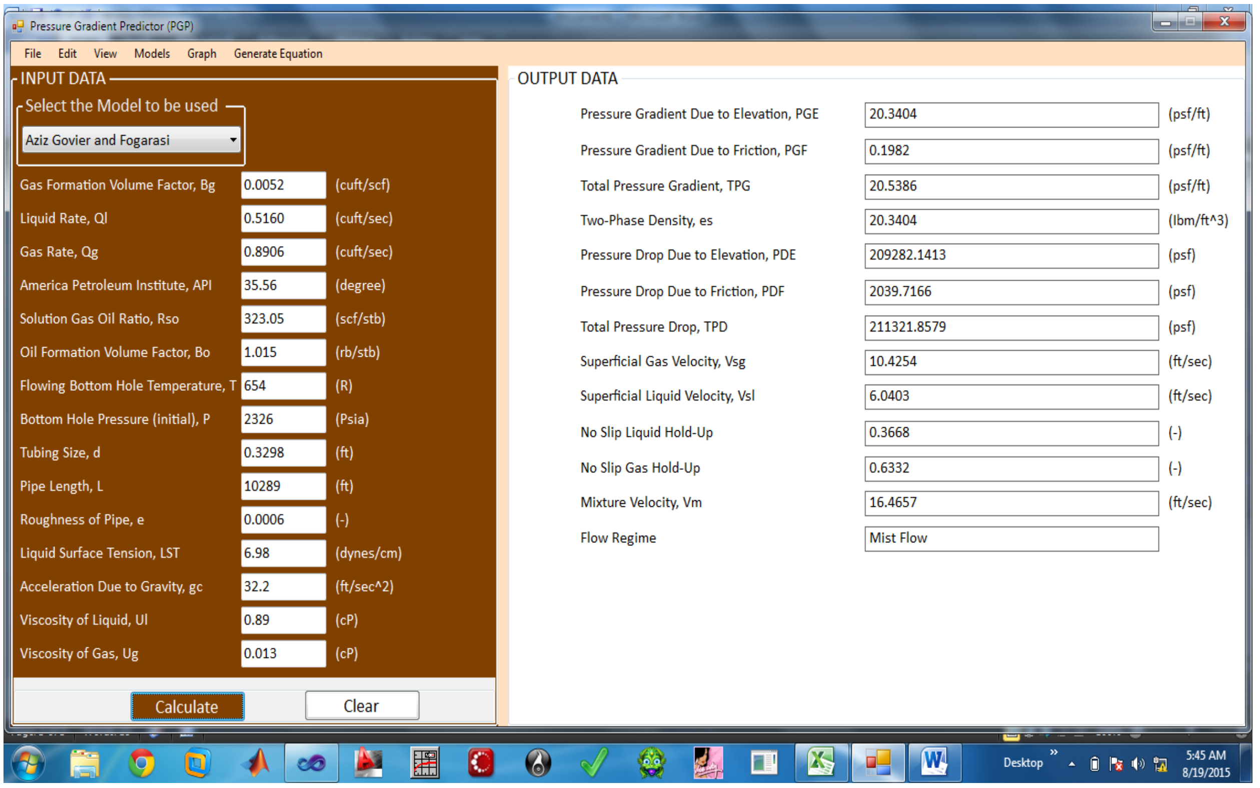

Figure 5 - analysis of well 2 with the Aziz et al model (for mist flow)

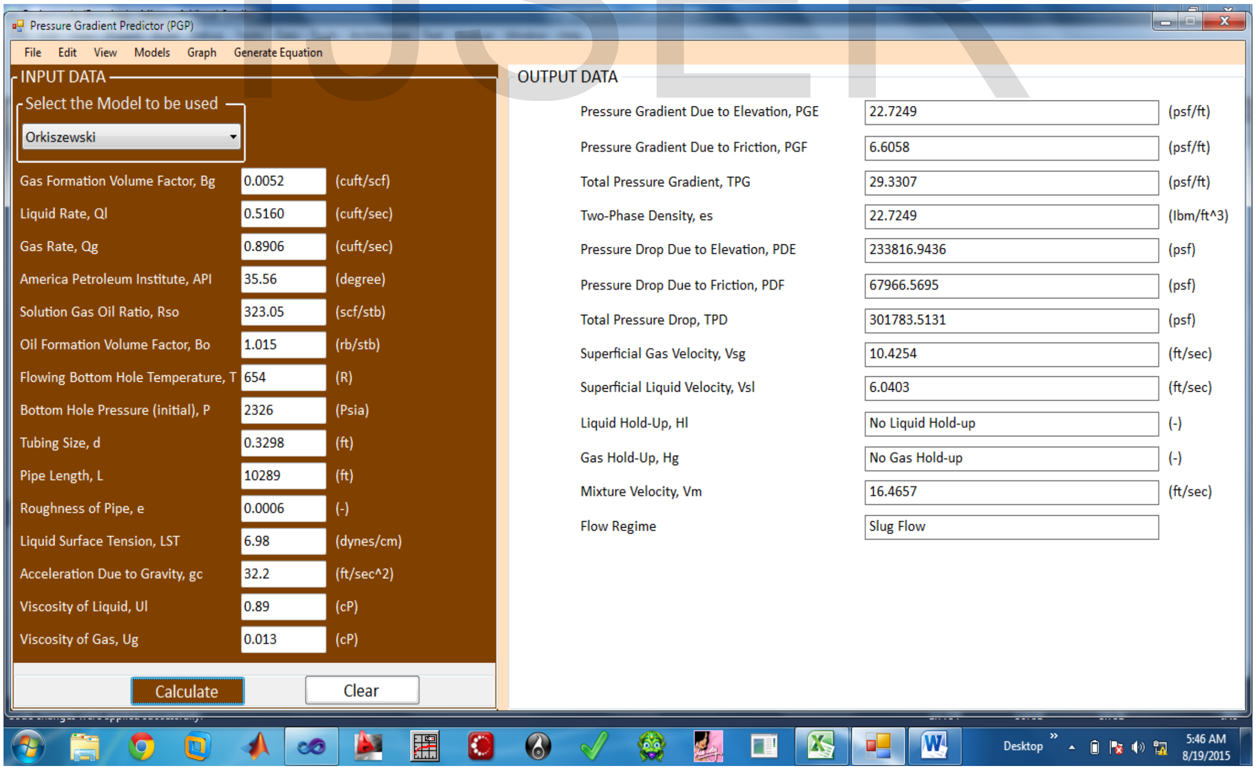

Figure 6 - analysis of well 2 with the Orkiszewski model (for slug flow)

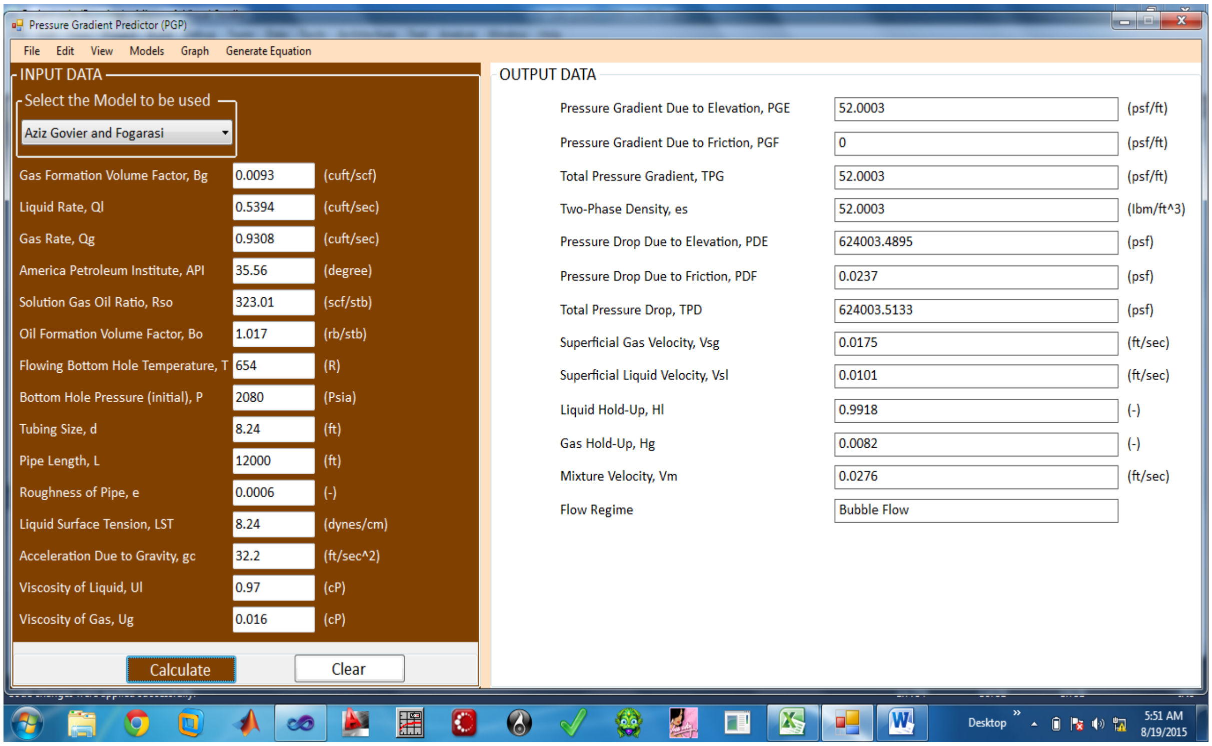

Figure 7 - analysis of well 3 with the Aziz et al model (for Bubble flow)

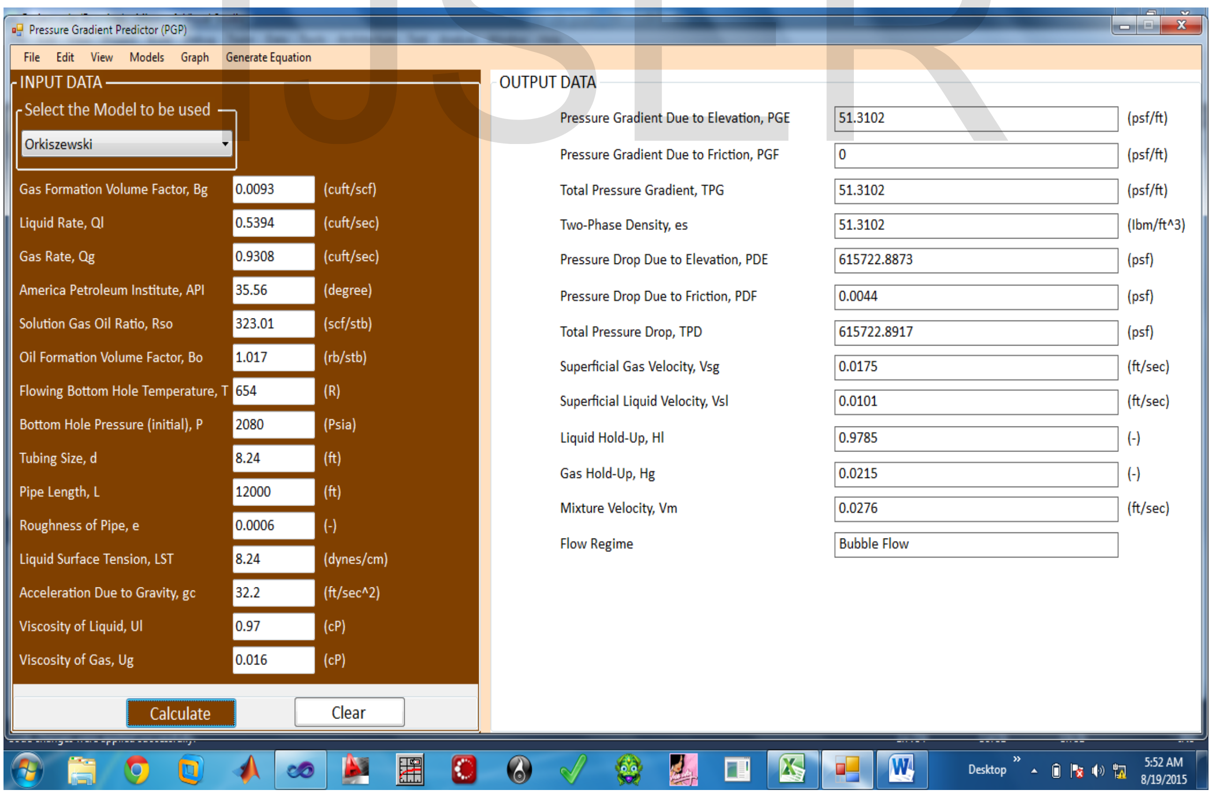

Figure 8 - analysis of well 3 with the Orkiszewski model (for Bubble flow)

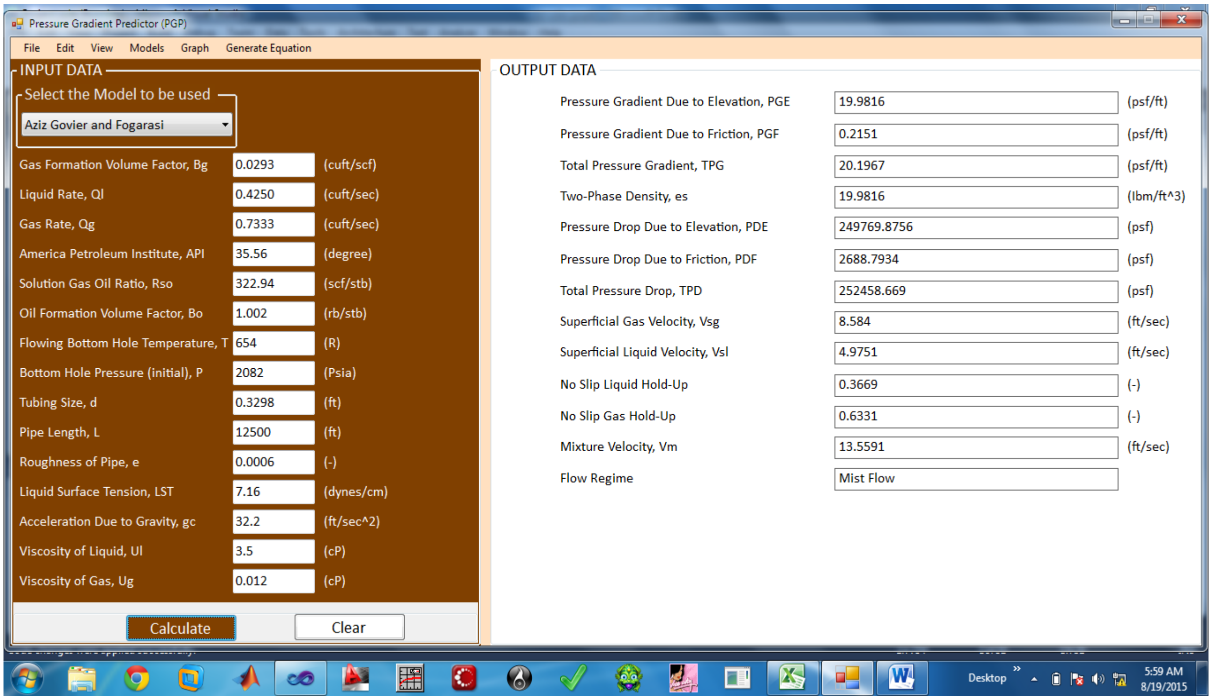

Figure 9 - analysis of well 4 with the Aziz et al model (for mist flow)

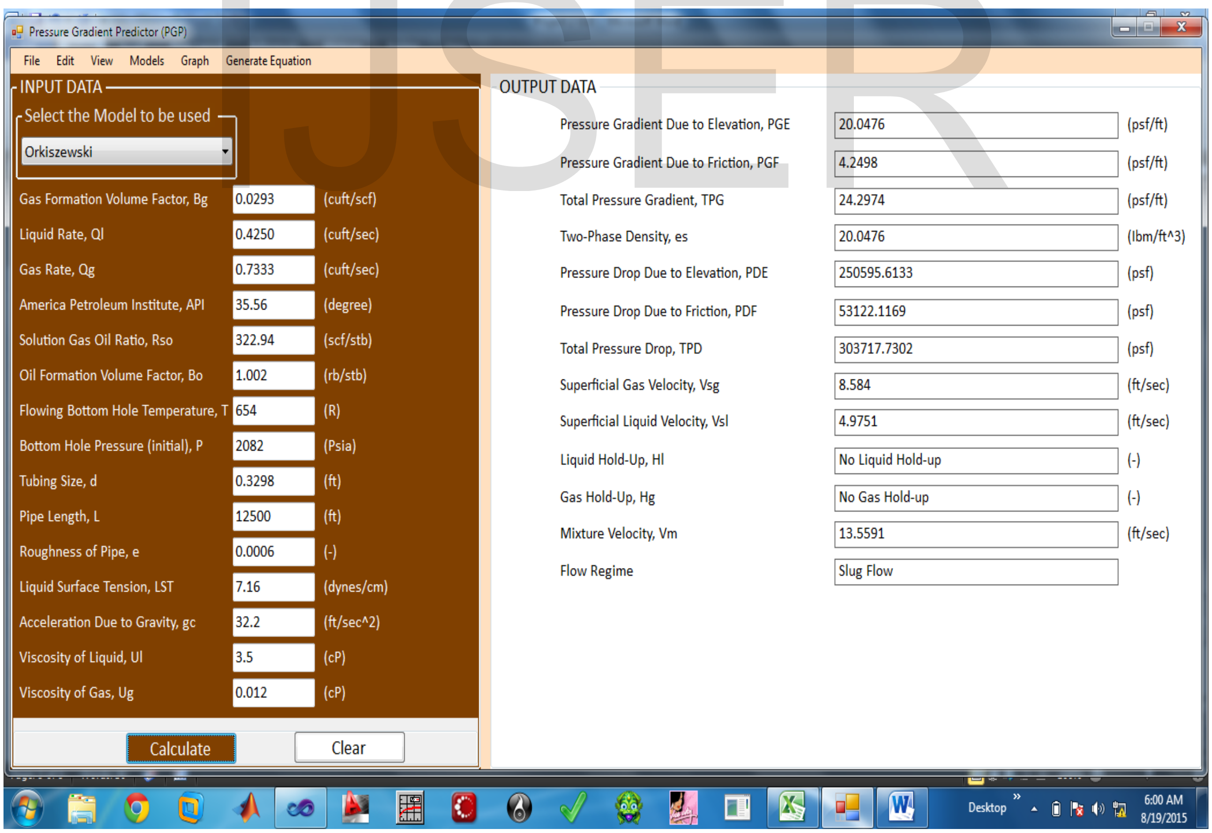

Figure 10 - analysis of well 4 with the Orkiszewski model (for slug flow)

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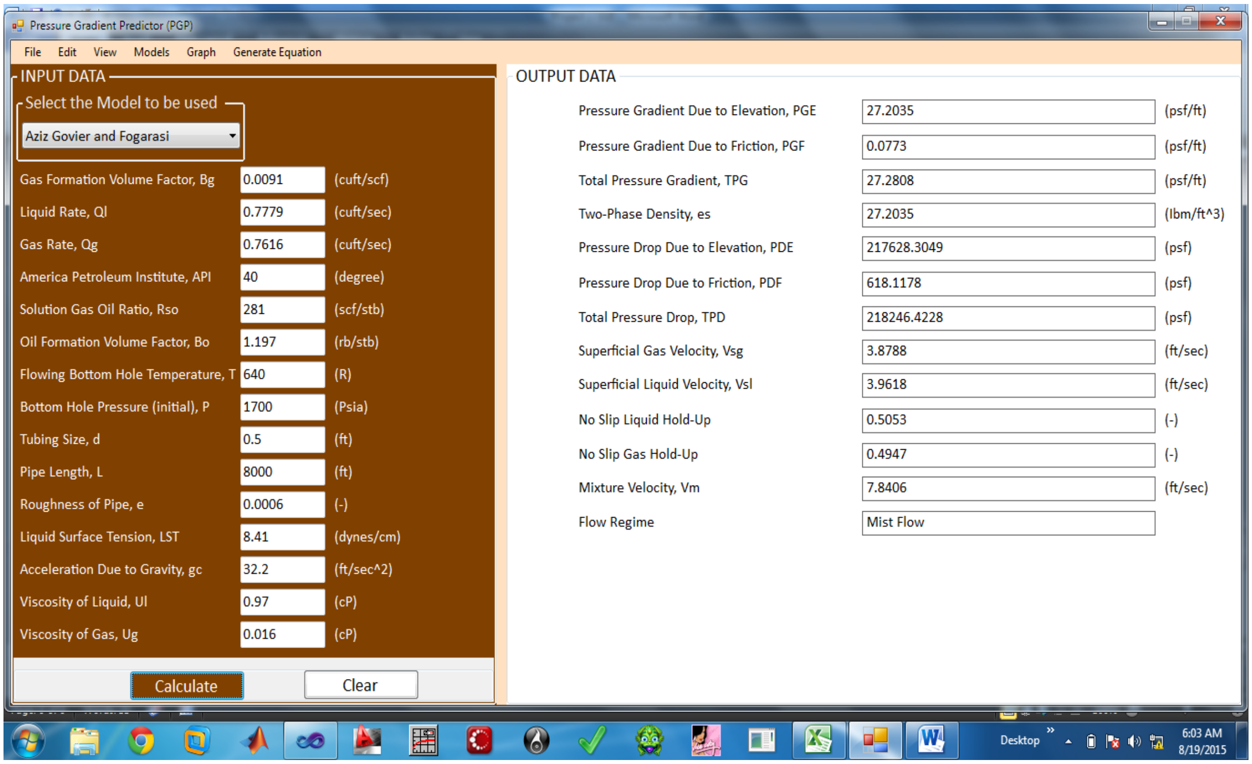


Figure 11 - analysis of well 5 with the Aziz et al model (for mist flow)

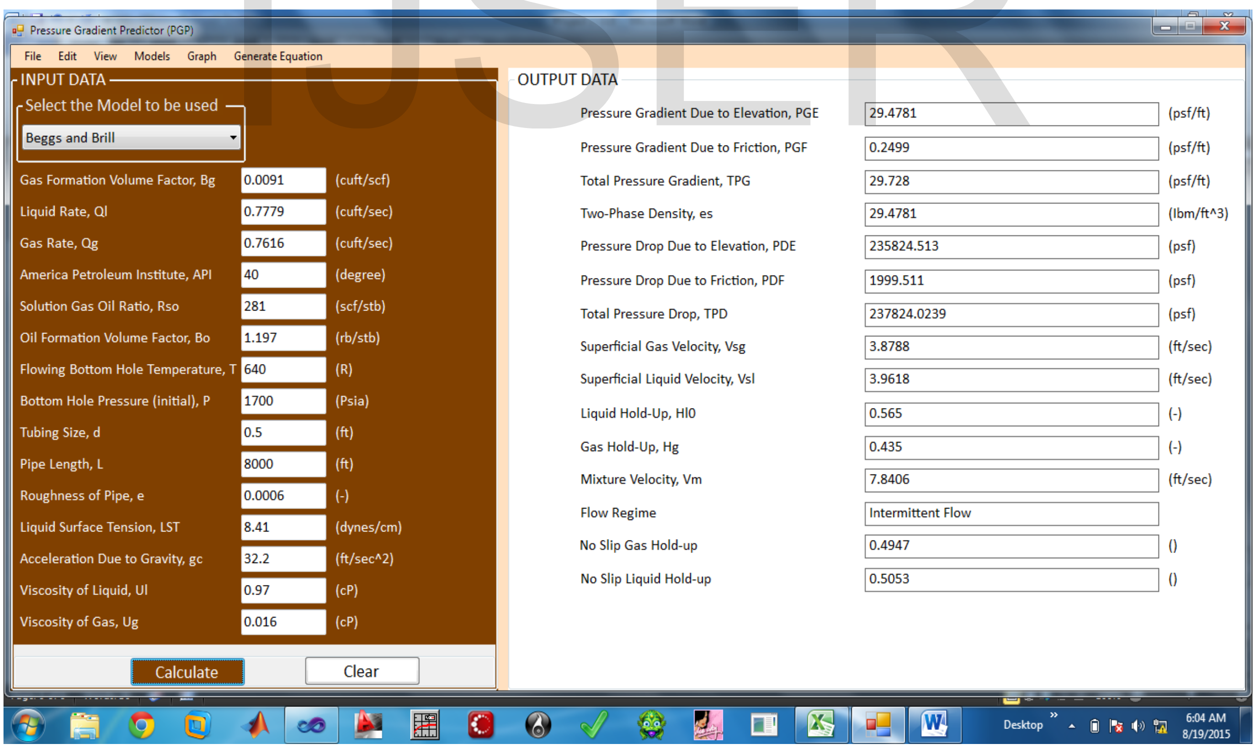


Figure 12 - analysis of well 5 with the Beggs and Brill model (for intermittent flow)

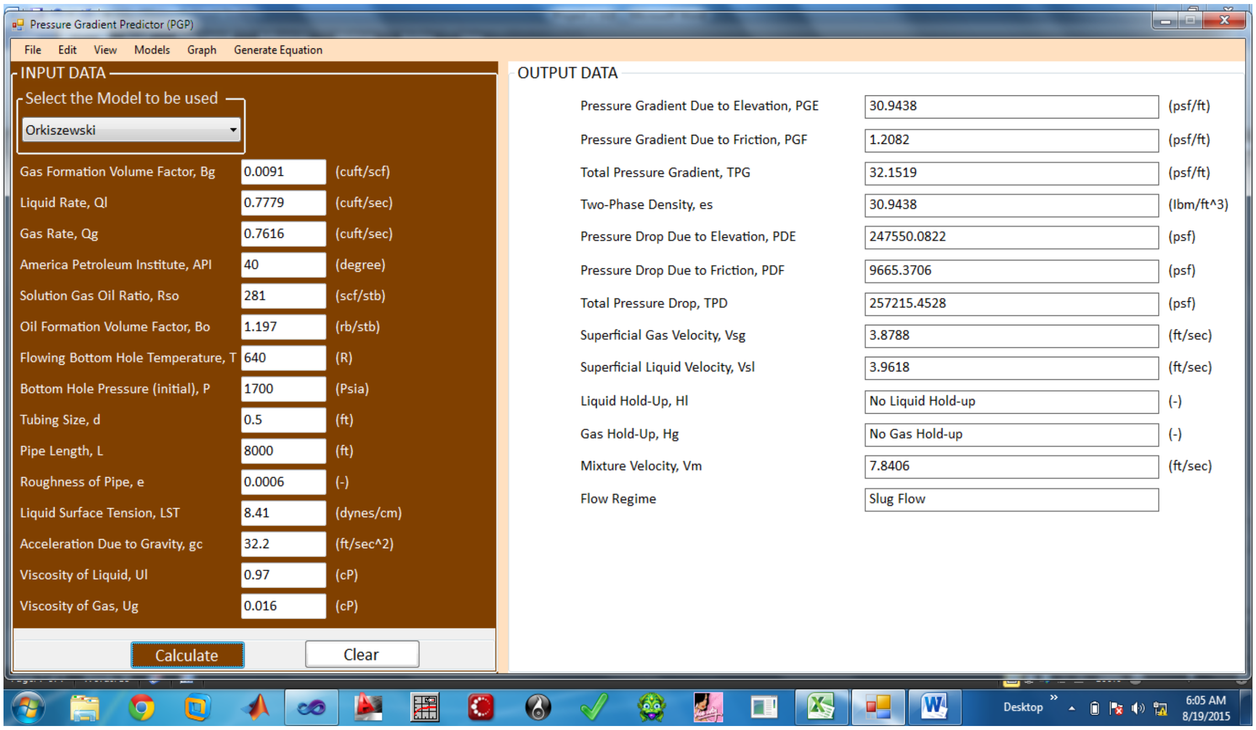


Figure 13 - analysis of well 5 with the Orkiszewski model (for slug flow)

The same analysis carried out for well 1 to well 5 as seen above, were also done for wells 6 to 11.

3.2 Discussion of results and validity of the Models

Figures 14 to 17 are representations on using an existing empirical correlation (Aziz et al. in [8], Beggs and Brill in [9], and Orkiszewski in [7]) on all the eleven wells. For mist flow, pressure gradient got from utilizing Aziz et al correlation for wells 1 to 7 were compared with actual field pressure gradients in the respective wells. From figure 14, wells 1 and 6 were almost accurately predicted with 0.79% and 0.69% errors respectively however well 2 is greatly under-predicted (30% error) probably due to the high age of the field. In contrast, wells 3,4,5,7 were slightly under-predicted with average percent error of 9%. The overall standard deviation was 0.0406. For Slug flow, pressure gradients got from utilizing the Orkiszewski's correlation in [7] for wells 1 to 7 were compared with the actual field pressure gradients in the respective wells. From figure 15, well 2 (0.68% error) was observed to be almost accurate while well 7 was under-predicted in contrast to wells 1,3,4,5,6 which were over-predicted with average percent error of 15.56%. The overall standard deviation was 0.0308. For intermittent flow, pressure gradients got from utilizing the Beggs and Brill's correlation in [9] for wells 1, 5. 6, 7, 8, 9, 10 and 11were compared with the actual pressure gradients in the respective wells. From figure 16, wells 1 and 5 were observed to be slightly under-predicted with percent errors of 7.77% and 3.30% respectively. In contrast, wells 6, 7, 8 were slightly over-predicted with percent errors of 2.73%, 4.44%, 4.04% respectively while wells 9, 10, 11 were over-predicted with average percent error of 14.99%. The overall standard deviation was 0.0278. For bubble flow, once again the Aziz et al correlation was utilized in estimating the pressure gradients for wells 8, 9, 10 and 11, these results were compared to the actual pressure gradients in the respective wells. From figure 17, wells 8, 9, 10, 11 were over-predicted with highest percent error of 16.9% occurring at well 8 due to the large tubing size (6'') because Pwf decreases as tubing size increases while wells 9, 10, 11 have an average percent error of 3.52%. The overall standard deviation was 0.0277. From the foregoing; the models developed performed satisfactorily for different flow regimes (using the empirical correlations as basis for establishing the regimes), having average percent error ranging from 6.87% (bubble flow), 8.41% (Intermittent flow), 9.79% (mist flow), to 12.13% (Slug flow) when compared (validated) against pressure gradient of each well measured at the field which is concordant with range recorded in literatures. It was observed that the pressure gradient was over-predicted (higher than measured) in many cases with few exceptions of under-prediction. However, excessive percent error noted in some wells could be as a result of any of the following factors; Assumptions made in developing the models such as neglecting both frictional and acceleration components, age of the well, fluid property correlations, water cut, and sanding etc.

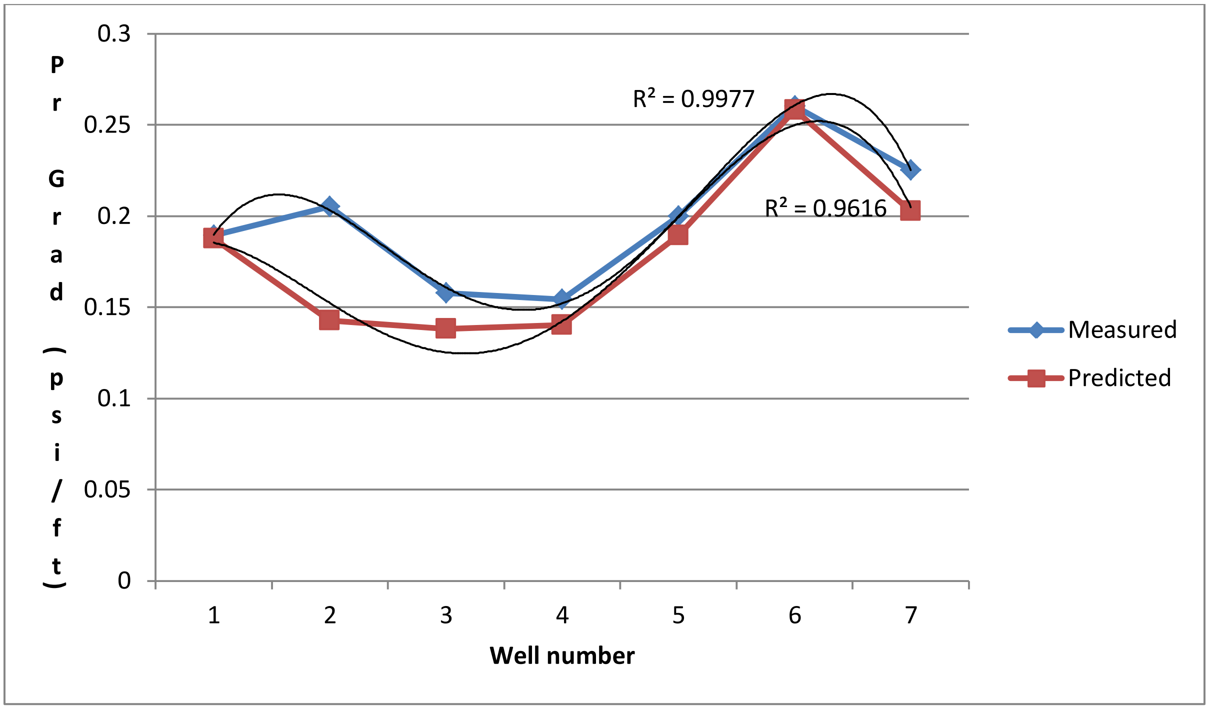
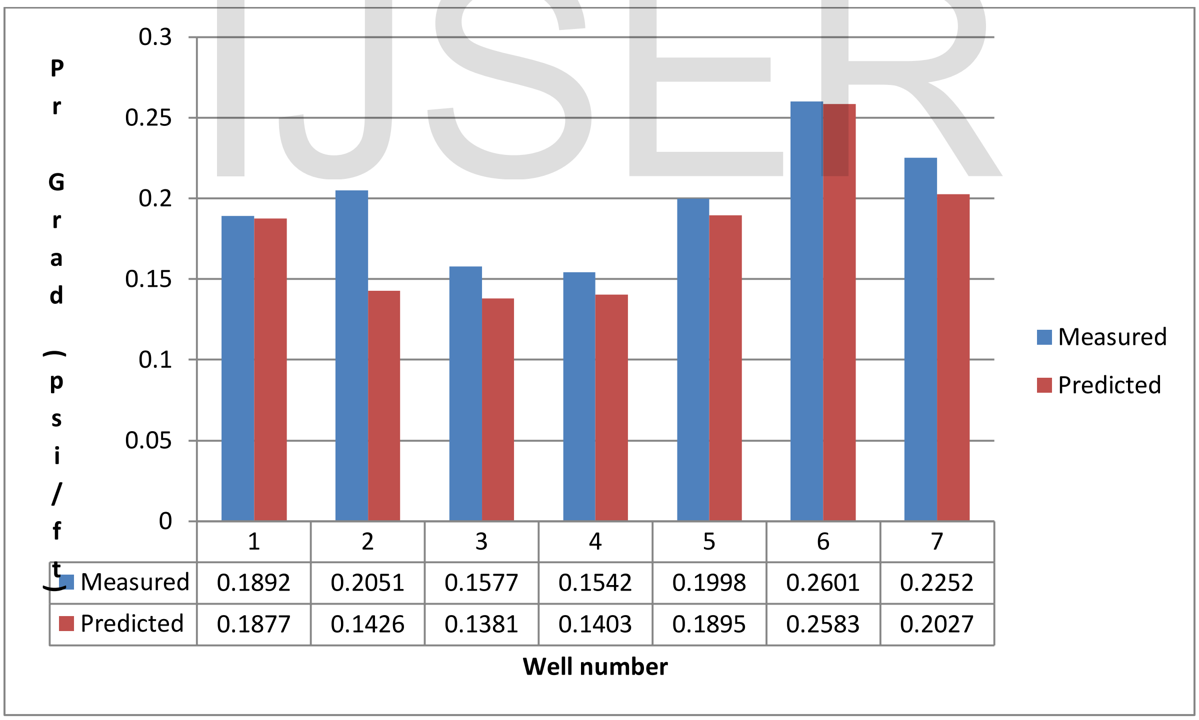


Figure 14(a) - Comparison of predicted PG and the measured PG (Mist flow)



	1	2	3	4	5	6	7
Measured	0.1892	0.2051	0.1577	0.1542	0.1998	0.2601	0.2252
Predicted	0.1877	0.1426	0.1381	0.1403	0.1895	0.2583	0.2027

Figure 14(b) - Comparison of predicted PG and the measured PG (Mist flow)

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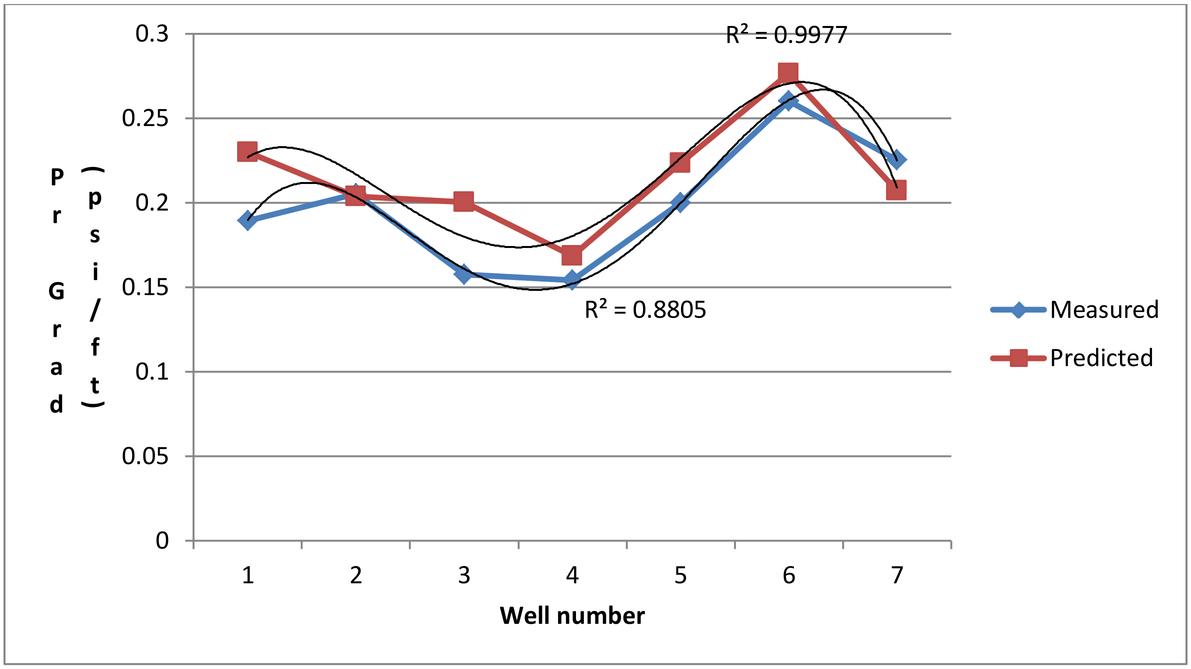
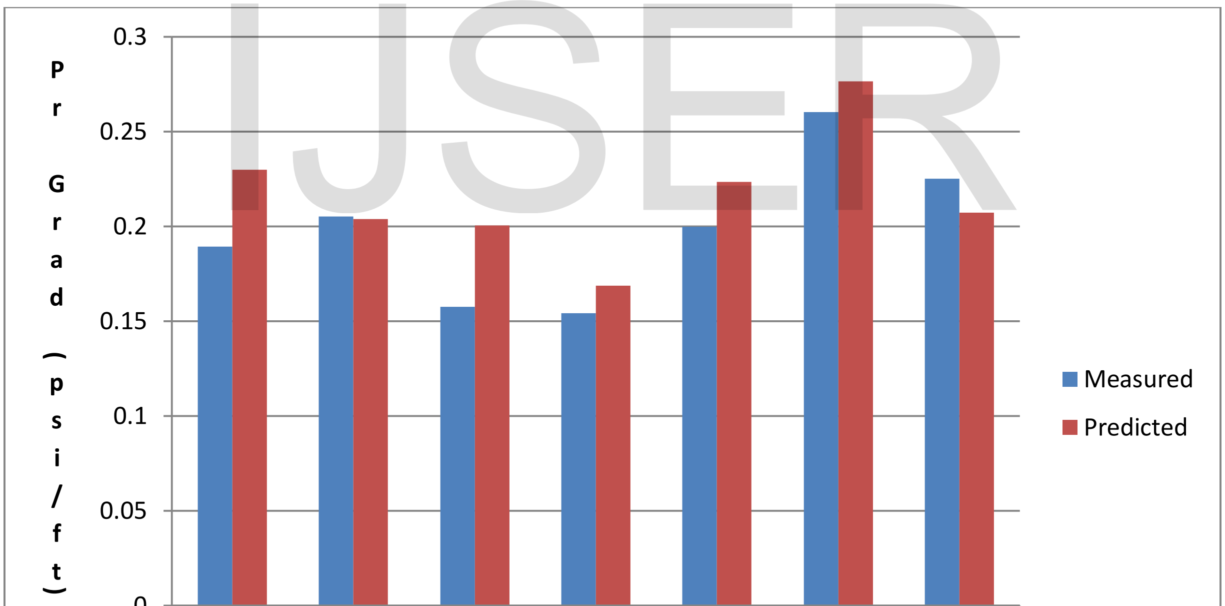


Figure 15(A) - Comparison of Predicted PG And The Measured PG (Slug Flow)



	1	2	3	4	5	6	7
Measured	0.1892	0.2051	0.1577	0.1542	0.1998	0.2601	0.2252
Predicted	0.2299	0.2037	0.2005	0.1687	0.2233	0.2766	0.2071

Well number

Figure 15(b) - Comparison of predicted PG and the measured PG (Slug flow)

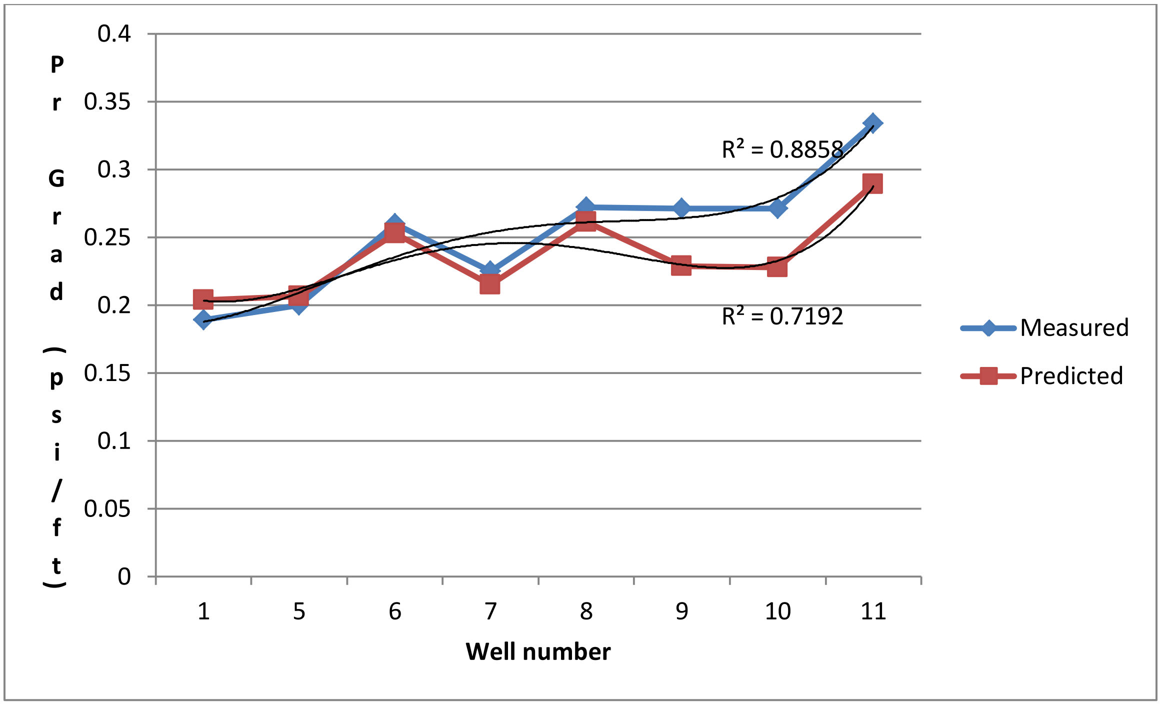
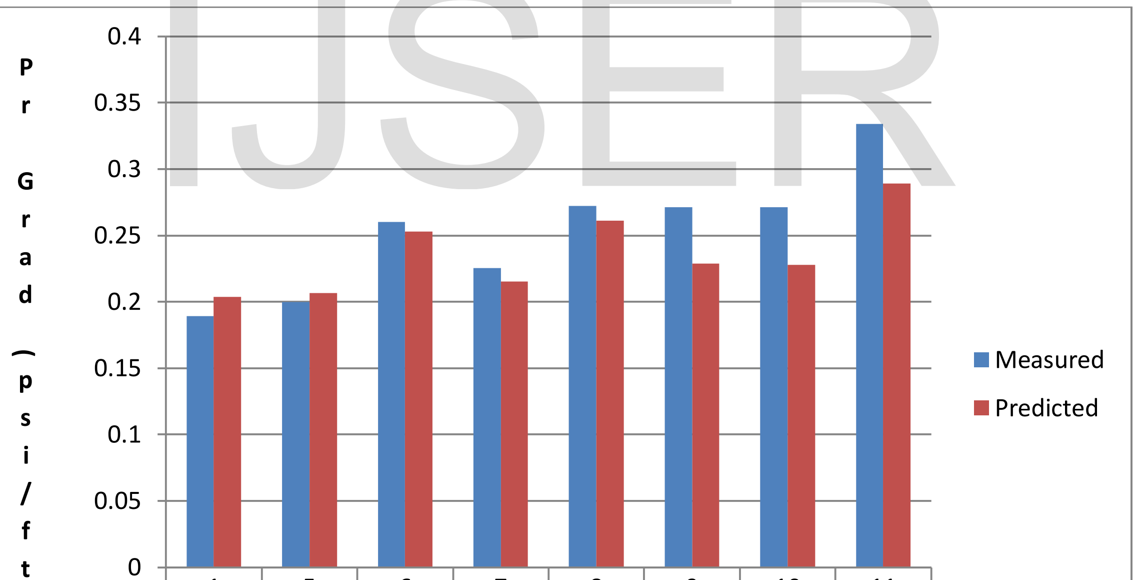


Figure 16(a) - Comparison of predicted PG and the measured PG (Intermittent flow)



	1	5	6	7	8	9	10	11
Measured	0.1892	0.1998	0.2601	0.2252	0.2721	0.2711	0.2713	0.334
Predicted	0.2039	0.2064	0.253	0.2152	0.2611	0.2289	0.228	0.2891

Well number

Figure 16(b) - Comparison of predicted PG and the measured PG (Intermittent flow)

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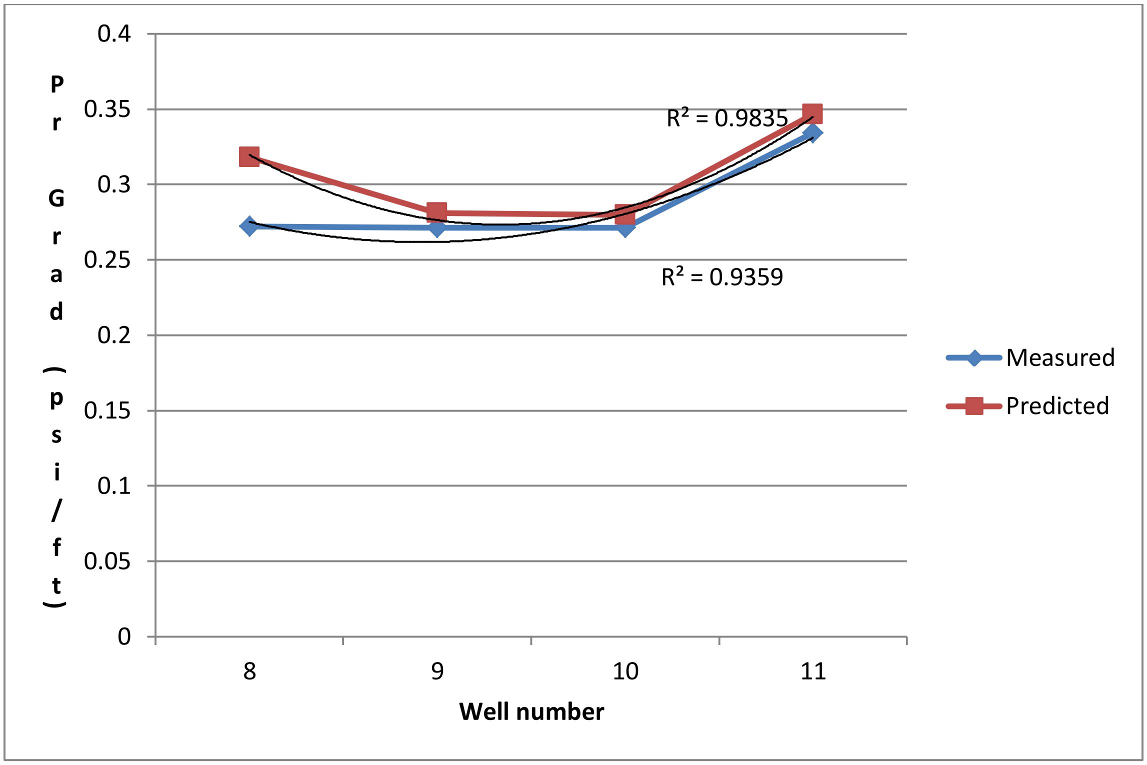

Figure 17(a) - Comparison of predicted PG and the measured PG (Bubble flow)

	8	9	10	11
Measured	0.2721	0.2711	0.2713	0.334
Predicted	0.3181	0.2811	0.2799	0.3464

Figure 17(b) - Comparison of predicted PG and the measured PG (Mist flow)

4 CONCLUSIONS

Based on this study, the following conclusions were made: Simple and quick-yield models for predicting pressure gradient in selected Niger Delta wells were developed, every well have a unique pressure gradient correlation indices which represent its propensity to having different flow regime at different stage of its life, the developed model is reliable as the average percent difference/error between observed and predicted PG agrees with that recorded in literature for other wells investigated in different regions, compared to existing models, the developed models yielded better results, being tailored for selected wells within the Niger Delta and finally, predictions with this model will be useful in: (i) Selecting correct tubing sizes; (ii) Envisaging when a well quit flowing & hence time for artificial lift; (iii) Designing artificial lift installations; (iv) Determining Pwf& PIs of wells; (v) Predicting maximum flow rates.

5 Recommendations

The following recommendations are suggested to highlight areas of additional research to improve the formulation of the model developed in this work:

i Increased number of producing well/field within the Niger-Delta should be used in both model formulation and development.

ii In order to improve the accuracy of two-phase flowing pressure gradient, efforts should be made to improve fluid property correlations.

iii Future study should incorporate horizontal and inclined wells.

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7 APPENDIX

7.1 General Multiphase Pressure gradient equation

$$\left[\frac{dP}{dZ}\right]_{Total} = \left[\frac{dP}{dZ}\right]_{elevation} + \left[\frac{dP}{dZ}\right]_{frictional} + \left[\frac{dP}{dZ}\right]_{acceleration} \qquad 1$$

But; $\left[\frac{dP}{dZ}\right]_{acceleration} = 0$ 2

7.2 Aziz et al Correlation

Bubble flow:

$$\left[\frac{dP}{dZ}\right]_{elevation} = \rho_L H_L + \rho_L(1 - H_L) \qquad 3$$

Where; $H_L = 1 - \frac{V_{sg}}{1.2V_m + V_{bs}}$ 4

$$V_{bs} = 1.4\left[\frac{\sigma_L g(\rho_L - \rho_g)}{\rho_L^2}\right]^{1/4} \qquad 5$$

$$\left[\frac{dP}{dZ}\right]_{frictinal} = \frac{f\rho_s V_m^2}{2g_c d} \qquad 6$$

Mist flow:

$$\left[\frac{dP}{dZ}\right]_{elevation} = \rho_L\frac{V_{SL}}{V_m} + \rho_g\frac{V_{sg}}{V_m} = \rho_m = \rho_L\lambda_L + \rho_g\lambda_g \qquad 7$$

$$\left[\frac{dP}{dZ}\right]_{frictional} = \frac{f\rho_g V_g^2}{2g_c d} \qquad 8$$

7.3 Beggs and Brill correlation

Intermittent flow:

$$\left[\frac{dP}{dZ}\right]_{elevation} = \rho_L H_L + \rho_g(1 - H_L) \qquad 9$$

Where; $H_L = H_L(\varphi)\boldsymbol{\psi}$ 10

$$\left[\frac{dP}{dZ}\right]_{frictional} = \frac{f_{tp}\rho_n V_m^2}{2g_c d} \qquad 11$$

$$\left[\frac{dP}{dZ}\right]_{total} = \frac{\left[\frac{dP}{dZ}\right]_{elevation} + \left[\frac{dP}{dZ}\right]_{frictional}}{1 - E_k} \qquad 12$$

$$E_k = \frac{\rho_L V_m V_{sg}}{g_c \rho} \qquad 13$$

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7.4 Orkiszewski Correlation

Slug flow:

$$\left[\frac{dP}{dZ}\right]_{elevation} = \frac{\rho_L(V_{SL}+V_b)+\rho_g V_{sg}}{V_m+V_b} + \rho_s\delta = \rho_s \qquad 14$$

$$\left[\frac{dP}{dZ}\right]_{frictional} = \frac{f\rho_L V_m^2}{2g_c d}\left[\frac{V_{SL}+V_b}{V_m+V_b} + \delta\right] = \rho_f \qquad 15$$

Bubble flow

$$\left[\frac{dP}{dZ}\right]_{elevation} = \rho_s = \rho_L H_L + \rho_g(1-H_L) \qquad 16$$

Where;

$$H_L = 1 - \frac{1}{2}\left[1 + \frac{V_m}{V_s} - \sqrt{\left(1 + {V_m}/{V_s}\right)^2 - 4\,{V_g}/{V_s}}\right] \qquad 17$$

$$\left[\frac{dP}{dZ}\right]_{frictional} = \frac{f\rho_L\left({V_{SL}}/{H_L}\right)^2}{2g_c d} \qquad 18$$

8 Nomenclature

ρ_L = Liquid density

d = Pipe Diameter

V_{sg} = Superficial gas velocity

λ_L = no-slip liquid holdup

q_L = Liquid flow rate

B_o = Oil formation volume factor

T = Temperature

μ_o = Oil Viscosity

H_L = Liquid holdup

V_m = Mixture Velocity

V_{SL} = Superficial liquid velocity

V_b = Bubble point velocity

λ_g = no slip gas holdup

q_g = gas flow rate

B_g = Gas formation volume factor

P = Pressure

μ_g = gas viscosity